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ACCELERATION OF PLASMOIDS IN WAVEGUIDES BY A SUPERHIGH-FREQUENCY WAVE

by

V. I. Domrin
R. R. Ramazashvili

(USSR)

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by V. I. Domrin &
R. R. Ramazashvili

SUMMARY

The acceleration is considered of an opaque plasmoid by an electromagnetic wave in a waveguide. The acceleration process is studied in detail for the case when the group velocity of the wave in the waveguide is much smaller than the speed of light in vacuum.

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The acceleration process of a plasma layer of finite thickness by a plane electromagnetic wave has been investigated in the work by Gurevich and Silin [1]. Since in this work the relativistic effects are not taken into account, the formulas obtained are valid only for nonrelativistic velocities of plasmoid motion.

In experiments plasmoids are usually accelerated in waveguides, in which the group velocity u of wave propagation may be significantly less than the propagation of light c in vacuum. Therefore, when applying this to the case of plasma acceleration in waveguides, the results of the work [1] will have to undergo certain changes. In particular, the low group wave propagation velocity in a waveguide will naturally limit the maximum plasmoid velocity attainable in the waveguide.

Let us consider a waveguide of infinite length, of which the dimensions and shape of cross-section are invariable along the axis. We shall consider that the axis of the waveguide coincides with the axis z. Assume that the plane $z = 0$ is the interface, to right of which is the plasma, and to the left the vacuum. Assume further that plasma is uniform and cold. Let an electromagnetic wave be incident upon the interface, with origin in the vacuum, and of which we shall consider the frequency ω as being much lower than the Langmuir frequency of the plasma, thus enabling us to neglect the penetration of the field into the plasma. We shall consider, moreover, that the interface between the field and the plasma remains plane in the course of the entire process.

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* USKORENIYA PLAZMENNYKH SGUSTKOV V VOLNOVODAKH SVCH VOLNOY

Such a system plasma-field shall be described by the laws of conservation [2]

$$\frac{\partial T_{ik}}{\partial x_k} = 0 \quad (i, k = 1, 2, 3, 4; \; x_4 = ict), \tag{1}$$

where T_{ik} is the system's energy-momentum tensor. Since we neglected the penetration of the field into the plasma, T_{ik} is equal to the sum of energy-momentum tensor of the electromagnet c field T_{ik}^f and of the energy-momentum tensor of particles T_{ik}^p

$$T_{\alpha\beta}^f = \frac{1}{4\pi}\left(-E_\alpha E_\beta - H_\alpha H_\beta + \frac{1}{2}(E^2 + H^2)\delta_{\alpha\beta}\right),$$

$$T_{4\alpha}^f = \frac{i}{4\pi}[\vec{E}\vec{H}]_\alpha, \quad T_{44}^f = -\frac{1}{8\pi}(E^2 + H^2), \quad (\alpha, \beta = 1, 2, 3) \tag{2}$$

$$T_{ik}^p = \mu c^2 u_i u_k.$$

Here μc^2 is the energy density of plasma's state of rest, and u_i is the macroscopic 4-velocity. When writing T_{ik}^p we neglected the mutual interaction of particles.

As is well known, two types of waves are possible in waveguides: the E-waves and the H-waves. The relationship of fields' transverse components $\vec{E}_\perp$ and $\vec{H}_\perp$ with the longitudinal components E_z and H_z in these waves is expressed by the following relations [3]

— in the E-wave

$$E_x = \frac{ik_z}{\varkappa^2}\frac{\partial E_z}{\partial x}, \quad E_y = \frac{ik_z}{\varkappa^2}\frac{\partial E_z}{\partial y},$$

$$H_x = -\frac{i\omega}{c\varkappa^2}\frac{\partial E_z}{\partial y}, \quad H_y = \frac{i\omega}{c\varkappa^2}\frac{\partial E_z}{\partial x}, \quad H_z = 0. \tag{3a}$$

— in the H-wave

$$E_x = \frac{i\omega}{c\varkappa^2}\frac{\partial H_z}{\partial y}, \quad E_y = -\frac{i\omega}{c\varkappa^2}\frac{\partial H_z}{\partial x}, \quad E_z = 0,$$

$$H_x = \frac{ik_z}{\varkappa^2}\frac{\partial H_z}{\partial x}, \quad H_y = \frac{ik_z}{\varkappa^2}\frac{\partial H_z}{\partial y}, \quad \varkappa^2 = \frac{\omega^2}{c^2} - k_z^2, \tag{3б}$$

$$E_z = E_0(x, y)\exp(-i\omega t + ik_z z), \quad H_z = H_0(x, y)\exp(-i\omega t + ik_z z).$$

Functions $E_0(x, y)$ and $H_0(x, y)$ are determined from equations

$$\Delta_\perp E_0(x, y) + \varkappa^2 E_0(x, y) = 0, \quad \Delta_\perp H_0(x, y) + \varkappa^2 H_0(x, y) = 0, \tag{4}$$

$$\left(\Delta_\perp \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$$

taking into account the conditions on the walls of the waveguide.

For definiteness we shall consider in the following only E-waves, but all the results obtained will be also valid for H-waves.

Let us integrate the system (1) over the waveguide's cross-section. Making use of boundary conditions for fields in a waveguide, we obtain the following system of equations:

$$\frac{\partial \int T_{44}\,ds}{\partial x_4} + \frac{\partial \int T_{43}\,ds}{\partial x_3} = 0,$$
$$\frac{\partial \int T_{34}\,ds}{\partial x_4} + \frac{\partial \int T_{33}\,ds}{\partial x_3} = 0. \tag{5}$$

The expressions for the components of the energy-momentum tensor depend on the velocity of the interface and on the velocity of particles reflected from it (because plasma is assumed to be cold, all the reflected particles will have an identical velocity). Therefore, we may state that under the above assumptions the system of equations (5) determined the motion velocity of the interface and of plasma particles. Averaging this system of equations in time, and taking advantage of the fact that the field does not penetrate into the plasma, we obtain

$$\int \overline{T_{43}{}^f}\,ds = \int \overline{T_{43}{}^p}\,ds, \quad \int \overline{T_{33}{}^f}\,ds = \int \overline{T_{33}{}^p}\,ds. \tag{6}$$

expressions, in which the line above denotes the averaging in time.

Let us denote by $\underline{v}$ the motion velocity of the interface and by v_1 that of particles reflected from the interface. In the system of counting, in which lies the interface, two groups of particles will be observed: particles incident upon the interface of which the velocity is $-\underline{v}$ and particles outflying from it, of which the velocity will be denoted by v_1'. Denoting the density of the number of particles by N, and the cross-section area of the waveguide by S, we have

$$\int \overline{T_{43}{}^p}\,ds = \frac{SNm}{2}\left(-\frac{v}{1-v^2/c^2} + \frac{v_1'}{1-v_1'^2/c^2}\right),$$
$$\int \overline{T_{33}{}^p}\,ds = \frac{SNm}{2}\left(\frac{v^2}{1-v^2/c^2} + \frac{v_1'^2}{1-v_1'^2/c^2}\right). \tag{7}$$

Here $\underline{m}$ is the mass of the ion (because of the smallness of m_e/m, the electronic part of the tensor $T_{ik}{}^p$ may be neglected).

In the above-made assumption about the total wave reflection from the interface, we shall obtain for the aggregate fields in the waveguide and in the interface system the following expressions:

$$E_z = 2E_0(x, y)\cos\omega't'\cos k_z'z',$$
$$E_x = -2\frac{k_z'}{\varkappa^2}\frac{\partial E_0}{\partial x}\cos\omega't'\sin k_z'z', \quad E_y = -2\frac{k_z'}{\varkappa^2}\frac{\partial E_0}{\partial y}\cos\omega't'\sin k_z'z', \tag{8}$$
$$H_x = -2\frac{\omega'}{c\varkappa^2}\frac{\partial E_0}{\partial y}\sin\omega't'\cos k_z'z', \quad H_y = 2\frac{\omega'}{c\varkappa^2}\frac{\partial E_0}{\partial x}\sin\omega't'\cos k_z'z',$$

where

$$\omega' = \frac{\omega - k_z v}{(1 - v^2/c^2)^{1/2}}, \quad k_z' = \frac{k_z - \omega v/c^2}{(1 - v^2/c^2)^{1/2}}.$$

When writing (8), we took advantage of the continuity condition of $\tilde{E}_{\perp}$ at the interface. Substituting (8) into (2), we obtain

$$\int \overline{T_{43}{}^{j}}\, ds = 0, \quad \int \overline{T_{33}{}^{j}}\, ds = \frac{k_z'^2}{4\pi\varkappa^2} \int E_0{}^2(x, y)\, ds. \tag{9}$$

Utilizing (7) and (9), we obtain from (2)

$$\frac{\int E_0{}^2(x, y)\, ds}{4\pi SNmc^2} \frac{(k_z - \omega v/c^2)^2}{\varkappa^2} = \frac{v^2}{c^2}. \tag{10}$$

Hence it is easy to find the interface's motion velocity

$$v = \frac{\alpha u}{\alpha + (1 - u^2/c^2)^{1/2}}. \tag{11}$$

Here $u = c^2 k_z / \omega$ is the group velocity of the wave in HP, and by α^2 we denoted the quantity

$$\alpha^2 = \int E_0{}^2(x, y)\, ds / 4\pi SNmc^2. \tag{12}$$

When the plasmas are not too rarefied, we always have $\alpha \ll 1$.

The motion velocity of charged particles in HP, v_1, is always easy to find from the realtivistic law of velocity addition

$$v_1 = \frac{2v}{1 + v^2/c^2}. \tag{13}$$

If the plasma has a finite thickness l, it is clear that in the time interval l/v all plasma particles will be imparted a velocity equal to v_1. Passing to the system moving with a velocity v_1, we again may utilize formulas (11)-(13), provided we understand by u the group velocity of the wave in the moving system. Pursuing further our discussion, we may investigate in detail the entire process of plasmoid acceleration in the waveguide.

The nonrelativistic case $u \ll c$ is the one that lends itself to the most complete investigation. In the limiting case, after one cycle of acceleration the velocity of the plasmoid is $2\gamma u$, $(\gamma \equiv \alpha/(1 + \alpha) < 1)$. In a system moving alongside with the plasmoid, the group velocity u of waves is

$$u_{(2)} = u(1 - 2\gamma), \tag{14}$$

and, after the second acceleration cycle the plasmoid velocity $v_{(2)}$ is

$$v_{(2)} = 2\gamma u(1-2\gamma). \tag{15}$$

Therefore after two acceleration cycles, v_2 becomes (in HP)

$$v_2 = 2\gamma u + 2\gamma u(1-2\gamma). \tag{16}$$

Generalizing this result for the case of n acceleration cycles, we obtain

$$v_n = 2\gamma u \sum_{m=0}^{n-1} (1-2\gamma)^m = u\,\{1-(1-2\gamma)^n\}. \tag{17}$$

It follows from this formula that in case of nonrelativistic group velocity of the waves, the maximum plasmoid velocity in the waveguide is exactly equal to u. In case of very small values of γ and finite n $(\gamma n \ll 1)$, (17) may lead to the form

$$v_n \approx 2n\gamma u, \tag{18}$$

showing that in this case the plasmoid velocity increases after each cycle by one and the same quantity $2\gamma u$ (cf. [1]).

It is not difficult to find the time necessary for the plasmoid to attain a specific velocity. Since for any specific m-th cycle the time interval

$$\tau_{(m)} = \frac{2l}{v_{(m)}} = \frac{l}{\gamma u(1-2\gamma)^{m-1}}, \tag{19}$$

is required, the following time will be necessary for the plasmoid to attain the velocity v_n

$$\tau_n = \sum \tau_{(m)} = \frac{l}{2\gamma^2 u}\,\frac{1-(1-2\gamma)^n}{(1-2\gamma)^{n-1}}. \tag{20}$$

As may be seen from (2), τ_n rises infinitely as $n \to \infty$. Therefore, the velocity of the plasmoid approaches the group wave velocity asymptotically. For $\gamma n \ll 1$ formula (20) takes the form

$$\tau_n \approx \frac{l}{\gamma u} n. \tag{21}$$

Let us find the distance the plasmoid covers after n acceleration cycles:

$$l_n = \sum_{m=1}^{n} (l + \tau_{(m)} v_{m-1}) = -\left(1 + \frac{\ln[(2\gamma^2 u/l)\tau_n + 1 - 2\gamma]}{|\ln(1-2\gamma)|}\right) l\,\frac{1-\gamma}{\gamma} + \tau_n u. \tag{22}$$

Hence it is easy to find the acceleration

$$\frac{d^2l_n}{d\tau_n^2}=\frac{4\gamma^3u^2(1-\gamma)}{l\,|\ln(1-2\gamma)|}\,\frac{1}{[(2\gamma^2u/l)\tau_n+1-2\gamma]^2}. \qquad (23)$$

Therefore the acceleration rapidly decreases with the rise of τ_n. In the case $\gamma n \ll 1$ (22) may be written in the form

$$l_n \approx n^2 l \approx \frac{\gamma^2u^2}{l}\tau_n^2, \qquad (24)$$

which shows that in this limit case the motion of the plasmoid is unifmormly accelerated.

We may find the energy accumulated by a single ion as a result of n cycles of acceleration

$$\varepsilon_n=\frac{1}{2}mv_n^2=\frac{2mu^4\gamma^4\tau_n^2}{[(1-2\gamma)l+2\gamma^2u\tau_n]^2}. \qquad (25)$$

This expression shows that the effective acceleration of the plasmoid takes place during a time of the order

$$\tau_{ef} \sim \frac{(1-2\gamma)l}{2\gamma^2u}. \qquad (26)$$

Utilizing (21) we may find the law of ε_n variation for $\gamma n \ll 1$:

$$\varepsilon_n \approx 2mu^2\gamma^2n^2 \approx 2mu^2\gamma^2(l_n/l). \qquad (27)$$

Therefore, at the beginning of the process the energy increases linearly with the distance. Then this dependence of ε_n on l_n weakens gradually, and it may be neglected at a distance of the order $\tau_{ef}\, u \sim (1-2\gamma)\, l/2\gamma^2$.

The formulas obtained by the above made assumptions fully describe the motion of plasmoids accelerated by a slow wave in the waveguide $(u \ll c)$. Comparison with the results of the work [1] shows that formulas (18), (21), (24) and (27), obtained in the assumption that $\gamma n \ll 1$, fully agree with the corresponding results of [1]. Naturally, a difference arises in the determination of the motion velocity of the boundary of the semi-infinite plasma in the waveguide (formula (11)). This difference is linked with the fact that the group velocity of plane waves in free space is c, while in the case considered this velocity may be much less than c. Contrary to the work [1], the assumption of smallness of group wave velocity allowed us to obtain formulas describing the motion of a plasmoid through its attainment of a velocity equal to the group velocity of waves.

When deriving the above general expressions, we did not impose any limitations on the quantity α. However, it is not difficult to see that the gradual acceleration of the plasmoid will take place only for $\alpha < 1$. As is seen from (14), the plasmoid, having attained the velocity $2\gamma u$ after the first acceleration cycle, will move more rapidly than the wave, and no subsequent acceleration will take place.

Let us now pass to the consideration of the relativistic case $u \sim c$. First of all we shall examine the case when the wave group velocity is so great that $\alpha \gg (1 - u^2/c^2)^{1/2}$. Then, as may be seen from (11), the velocity of the boundary will already be equal to the wave group velocity after one acceleration cycle. As to the plasmoid velocity, it will be

$$v = \frac{2u}{1 + u^2/c^2} \sim u \sim c. \tag{28}$$

(It is clear that the same pattern will take place in the case $\alpha \gg 1$ regardless of the magnitude of the group velocity. The velocity of the plasmoid will be equal to $2u/(1 + u^2/c^2)$. It is easy to see that for real fields and plasmas the conditions $\alpha \gg 1$ and $\omega^2 \ll \omega_p^2$ are difficult to realize simultaneously).

Most complex for the investigation is the inverse limiting case $\alpha \ll (1 - u^2/c^2)^{1/2}$. One may not succeed in obtaining for such a case a compact formula determining the velocity of the plasmoid in HP after an arbitrary number of plasmoid acceleration cycles. However, the initial acceleration cycle processes may be described with good precision for the nonrelativistic case by the formulas above. The fact is that, because of the smallness of $\alpha/(1 - u^2/c^2)^{1/2}$ at the beginning of the process, the plasmoid velocity will remain nonrelativistic till the completion of a specific number of cycles, as this may be seen from (11). Certain complications arise because the denominator of (11) contains $(1 - u^2/c^2)^{1/2}$ and consequently changes in the process of plasmoid acceleration. However, for initial cycles we may apparently consider the group velocity as invariable in the denominator without committing great error, and thus utilize the above formulas in which γ is recognized for the expression $\alpha/(1 - u^2/c^2)^{1/2}$. Such a consideration will be valid until the plasmoid attains the relativistic velocity.

It should be borne in mind that the above assumptions may limit the applicability of the formulas obtained to a real experiment on plasmoid acceleration in waveguides. It is clear that the assumption of invariability of the interface-field-plasma is not in any way evident because of waveguide field dependence on transverse coordinates. However, cases may be indicated when this assumption may be fulfilled with good precision. For example, as is well known [4], plasmoids of specific configurations may fully close the waveguides in conditions when their dimensions are small by comparison with the wavelength. Under these conditions the field inhomogeneities at distances of the order of plasmoid dimensions may be neglected and the acceleration process may be described by the above formulas.

Let us point also to one limitation that was discreetly assumed in the work. Formula (11) describes a stationary picture. However, prior to reaching the stationary velocity the plasma interface will shift by a specific distance, which for the validity of our reasonings should be much smaller than the dimensions of our plasmoid. We considered this distance to be of the order of skin-layer thickness and we neglected it completely. However, this question calls for thorough investigation. A more detailed research is also deserved by the questions of field penetration into plasma and its absorption and scattering, for the field absorption may lead to plasma heating, which is going to complicate the pattern of plasmoid acceleration described here.

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**** THE END ****

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1145 19th St. NW
D. C. 20036; tel: 223-6700; 223-4930.

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